

Reg. No. : .....

Name : .....

**Third Semester M.Sc. Degree Examination, February 2024**

**Mathematics**

**Elective II**

**MM 234 : GRAPH THEORY**

**(2015 – 2016 Admissions)**

Time : 3 Hours

Max. Marks : 75

- Instructions: (1) All questions carry equal marks.  
(2) Answer either Part A or Part B of each questions.

1. (A) (i) Draw all non-isomorphic graphs having 4 vertices.  
(ii) Prove that isomorphism is an equivalence relation on the set of all graphs.  
(iii) 'Every non trivial connected graph contains at least two vertices which are not cut vertices'. State True or False and Justify.

OR

- (B) (i) Define Self complementary graphs. Give an example. Prove that the number of vertices  $n$  in a self-complementary graph  $G$  satisfy either  $n \equiv 0 \pmod{4}$  or  $n \equiv 1 \pmod{4}$ .  
(ii) Determine  $Aut(C_5)$ .  
(iii) Define  $k(G), \lambda(G)$ . Find these values for  $K_n$ . State and prove any result connecting these.

P.T.O.



2. (A) (i) Define Eulerian graph. Give an example of an Eulerian graph.
- (ii) Prove that a graph is Eulerian if and only if every vertex of  $G$  is Eulerian.

OR

- (B) (i) Define Hamiltonian graph. Prove that the Petersen graph is non Hamiltonian.
- (ii) Define closure of a graph. Prove that a graph  $G$  is Hamiltonian if and only if its closure is Hamiltonian.

3. (A) (i) State and prove first theorem of Digraph Theory.
- (ii) Prove that every tournament contains a Hamiltonian path.

OR

- (B) (i) State and prove Marriage theorem.
- (ii) Prove that the Petersen graph is not 1 factorable.

4. (A) (i) Briefly explain Four color problem and state the Four color theorem.
- (ii) Determine the Chromatic number of the Petersen Graph and the wheel graph.
- (iii) Draw a triangle free graph with chromatic number 4.

OR

- (B) (i) State and prove Vizing's theorem.
- (ii) Find  $r(P_3, K_3)$



5. (A) (i) Prove that every graph is the center of some graph.
- (ii) Find the radius, diameter and center of  $P_n, C_n$ .

OR

- (B) (i) Prove that the detour distance is a metric on the vertex set of every connected graph.
- (ii) Define radio coloring and radio  $k$  chromatic number with proper examples.

**(5 × 15 = 75 Marks)**

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