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Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree Examination, September 2024

Mathematics

MM 523 : PARTIAL DIFFERENTIAL EQUATIONS AND INTEGRAL EQUATIONS

(2023 Admission)

Time : 3 Hours Max. Marks : 75

PART – A

Answer any **five** questions. Each question carries **3** marks.

1. When do you say that a problem is well-posed?

2. What is the general form of a second order quasi-linear equation in two variables?

3. Explain the method of characteristics.

4. Find a canonical transformation $q = q(x, y), r = r(x, y)$ for the differential equation $u_{xx} + xu_{yy} = 0$.

5. What is the difference between Fredholm linear integral equations and volterra linear integral equations?

6. Explain the variational interation method for solving Fredholm integral equations.

7. Find $\frac{d}{dx}$ for the integral $\int_0^{x^2} e^{xt} dt$ using Leibriz rule.

8. Convert the initial value problem $y' + y = 0$, $y(0) = 1$ to a volterra integral equation.

(5 × 3 = 15 Marks)

P.T.O.

PART – B

Answer **all** questions. Each question carries **10** marks.

9. (A) (a) Solve the equation $u_x + 3y^{2/3}u_y = 2$ subject to the initial condition $u(x, 1) = 1 + x$. **5**

(b) Consider the equation $u_x + u_y = 1$ with the initial condition $u(x, 0) = f(x)$

(i) What are the projections of the characteristic curves on the (x, y) plane?

(ii) Solve the equation. **5**

OR

(B) (a) Solve the problem

$$xu_y - yu_x + u = 0$$

$$u(x, 0) = 1, \quad x > 0.$$

Is the solution unique? What is the maximal domain where it is defined? **5**

(b) Find at least five solutions for the cauchy problem

$u_x + u_y = 1$, $u(x, x) = x$. **5**

10. (A) (a) Prove that the type of a linear second-order PDE in two variables is invariant under a change of coordinates. **8**

(b) Prove that the equation

$x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} + xu_x + yu_y = 0$ is parabolic. **2**

OR

(B) (a) Consider the equation

$$xu_{xx} - yu_{yy} + \frac{1}{2}(u_x - u_y) = 0$$

(i) Find the domain where the equation is elliptic, and the domain where it is hyperbolic.

(ii) For each of the domains, find the corresponding canonical transformation. **5**

(b) Consider the equation

$$u_{xx} - 2u_{xy} + 4e^y = 0.$$

Find the canonical form of the equation. **5**

11. (A) (a) Prove the cauchy problem:

$u_{tt} - c^2 u_{xx} = 0\,,\ -\infty < x < \infty\,,\ t > 0$

$u(x,\,0) = f(x),\ u_t(x,\,0) = g(x),\ -\infty < x < \infty$

is ill-posed. On the domain $-\infty < x < \infty$ **5**

(b) Prove that the cauchy problem

$u_{tt} - c^2 u_{xx} = F(x,\,t),\ -\infty < x < \infty, t > 0$

$u(x,\,0) = f(x), u_t(x,\,0) = g(x),\ -\infty < x < \infty$

admits at most one solution. **5**

OR

(B) (a) Solve the following cauchy problem

$u_{tt} - 9u_{xx} = e^x - e^{-x},\ -\infty < x < \infty, t > 0$

$u(x,\,0) = x,\ -\infty < x < \infty$

$u_t(x,\,0) = \sin x,\ -\infty < x < \infty.$ **5**

(b) Solve the Heat equation

$u_t - ku_{xx} = 0,\ 0 < x < L,\ t > 0$

$u(0,\,t) = u(L,\,t) = 0,\ t \geq 0$

$u(x,\,0) = f(x),\ 0 \leq x \leq L$

where f is a given initial condition, by the method of separation of variables. **5**

12. (A) (a) Convert the volterra integral equation

$$u(x) = x - \cos x + \int_0^x (x - t)u(t)dt$$

to an equivalent initial value problem. **5**

(b) Derive an equivalent volterra integral equation to the following initial value problem:

$y'' + y = \sin x,\ y(0) = 0,\ y'(0) = 0\,.$ **5**

OR

(B) (a) Drive the equivalent Fredholm integral equation of the following boundary value problem:

$y'' + 4y = \sin x,\ 0 < x < 1,\ y(0) = y(1) = 0\,.$ **5**

(b) Convert the following initial value problem to an equivalent volterra integral equation

$y''' - y'' - y' + y = 0,\ y(0) = 2,\ y'(0) = 0,\ y''(0) = 2$ **5**

13. (A) (a) Solve the Fredholm integral equation by using the noise term phenomenon

$$u(x) = x\cos x + 2x + \int_0^{\pi} xu(t)dt\,. \qquad \textbf{5}$$

(b) Solve the following Fredholm integral equation by using the vairational interation method:

$$u(x) = e^{-x} + 2x + \frac{3}{2}\int_{-1}^{0} xtu(t)dt\,. \qquad \textbf{5}$$

OR

(B) (a) Solve the following Fredholm integral equation by using the direct computation method:

$$u(x) = x^2 - \frac{25}{12}x + 1 + \int_0^1 xt\,u(t)dt\,. \qquad \textbf{5}$$

(b) Solve the following Fredholm integral equation by using the successive approximate method

$$u(x) = \sin x + \int_0^{\pi/2} \sin x \cos t\, u(t)dt\,. \qquad \textbf{5}$$

14. (A) (a) Use the variational iteration method to solve the following volterra integral equation:

$$u(x) = 3x - 9\int_0^x (x-t)u(t)dt\,. \qquad \textbf{5}$$

(b) Solve the following volterra integral equation by using the series solution method:

$$u(x) = 1 + x - \frac{2}{3}x^3 - \frac{1}{2}x^4 + 2\int_0^x t\,u(t)dt\,. \qquad \textbf{5}$$

OR

(B) (a) Solve the following volterra integral equation by converting to equivalent initial value problem:

$$u(x) = x + \frac{x^3}{3!} - \int_0^x (x-t)u(t)dt\,. \qquad \textbf{5}$$

(b) Find the solution of the volterra equation of the first kind

$$x^2 + \frac{x^3}{6} = \int_0^x (2+x-t)u(t)dt\,. \qquad \textbf{5}$$

(6 × 10 = 60 Marks)
