

(Pages : 3)

T – 6377

Reg. No. :

Name :

Second Semester M.Sc. Degree Examination, September 2024

Mathematics

MM 524 : ADVANCED TOPOLOGY

(2023 Admission)

Time : 3 Hours

Max. Marks : 75

PART – A

Answer any **five** questions. Each question carries **3** marks.

1. If $(x_\lambda : \lambda \in \Lambda)$ is a net in a space X and $x_\lambda \rightarrow x \in X$, then show that for any subnet $(x_{\lambda_\sigma} : \sigma \in \Sigma)$ we have $x_{\lambda_\sigma} \rightarrow x$.
2. Show that the projection $\pi_1 : \mathbb{R}^2 \rightarrow \mathbb{R}$ is not a closed mapping.
3. Define T_0, T_1 and T_2 -spaces.
4. Show that a closed subspace of a normal space is normal.
5. Define a directed set.
6. Let X be a space, Y a T_2 -space and $f, g : X \rightarrow Y$ continuous functions. If f and g agree on a dense subset of X , then show that $f = g$.
7. What is meant by homotopy between paths?
8. What is a deformation retraction?

(5 × 3 = 15 Marks)

P.T.O.



PART – B

Answer **all** questions. Each question carries **10** marks.

9. A. (a) Suppose $f: X \rightarrow Y$ is a continuous mapping onto Y . Show that the mapping f is closed if and only if whenever $y \in Y$ and U is an open subset of X with $f^{-1}(y) \subset U$, there is an open set $V \subset Y$ with $f^{-1}(y) \subset f^{-1}(V) \subset U$. **6**

(b) Suppose $f: X \rightarrow Y$ is a quotient mapping onto Y . Show that a mapping $g: Y \rightarrow Z$ is continuous if and only if $g \circ f: X \rightarrow Z$ is continuous. **4**

OR

B. (a) If $f: X \rightarrow Y$ is continuous and onto Y , then show that f is a quotient mapping if f is either open or closed. **8**

(b) Show that the product topology on $\prod_{\alpha \in A} X_{\alpha}$ is the weakest topology that makes all projections continuous. **2**

10. A. If X is a topological space then show that the following are equivalent:

(a) X is compact.

(b) If \mathcal{F} is a collection of closed subsets of X with the finite intersection property then $\bigcap \mathcal{F} \neq \emptyset$.

(c) Every filter on X has a cluster point.

(d) Every ultrafilter on X converges. **10**

OR

B. (a) Show that every filter is contained in an ultrafilter. **6**

(b) Show that a filter \mathcal{F} on a set X is an ultrafilter on X if and only if for every $A \subset X$, either $A \in \mathcal{F}$ or $X \setminus A \in \mathcal{F}$. **4**



11. A. (a) Show that a T_1 -space X is regular if and only if for each point a in X and each open set U containing a , there is an open set W containing a whose closure is contained in U . **6**
- (b) Show that a simply ordered set with its order topology is a Hausdorff space. **4**

OR

- B. (a) Show that a T_1 -space X is regular if and only if for each point a in X and closed set C not containing a , there exist open sets U and V in X such that $a \in U$, $C \subset V$ and \bar{U} and \bar{V} are disjoint. **5**
- (b) Show that the product of any family of regular spaces is regular. **5**
12. A. (a) Show that every metric space is normal. **5**
- (b) Show that the Sorgenfrey plane is regular but not normal. **5**

OR

- B. (a) State and prove the Urysohn's Lemma. **6**
- (b) Show that every completely regular space is regular. **4**
13. A. State and prove the Covering Path Property. **10**

OR

- B. (a) Define equivalence of paths and show that the relation of equivalence for paths is an equivalence relation. **6**
- (b) Show that every contractible space is simply connected. **4**
14. A. Let X_1 and X_2 be spaces with base points x_1 and x_2 respectively. Then show that $\Pi_1(X_1 \times X_2, (x_1 \times x_2)) \cong \Pi_1(X_1, x_1) \otimes \Pi_1(X_2, x_2)$. **10**

OR

- B. Prove that for $n \geq 2$, the n -sphere S^n is simply connected. **10**

(6 × 10 = 60 Marks)

