(Pages : 3) **T – 6371**

**Reg. No. : .............................**

**Name : ..................................**

## Second Semester M.Sc. Degree Examination, September 2024

## Mathematics

## MM 222 : REAL ANALYSIS II

## (2020-2022 Admission)

Time : 3 Hours Max. Marks : 75

PART – A

Answer any **five** questions. Each question carries **3** marks.

1. Show that outer measure is translation invariant.

2. Show that the derivatives of a continuous function are measurable.

3. Give an example of a non-unique extension of a non $\sigma$-finite measure.

4. Define convex and strictly convex functions and give an example.

5. Prove that a countable union of sets, positive with respect to a signed measure $v$ is a positive set.

6. If $c$ is a real number and $f$ is a measurable function, prove that $f+c$ is measurable.

7. Let $f$ be the function defined by $f(0)=0$ and $f(x)=x\sin\left(\frac{1}{x}\right)$. Find $D^{+}f(0)$ and $D_{-}f(0)$.

8. Show that monotone functions are measurable.

**($5 \times 3 = 15$ Marks)**

P.T.O.

PART – B

Answer **all** questions. Each question carries **12** marks.

9. (A) (a) Let $T$ be measurable set of positive measure and let $T^* = [x - y : x, y \in T]$. Show that $T^*$ contains an interval $(-\alpha, \alpha)$ for some $\alpha > 0$. **8**

(b) Prove that every interval is measurable. **4**

OR

(B) (a) Prove that the class $\mathscr{M}$ is a $\sigma$ - algebra. **8**

(b) Let $f$ be a measurable function and $B$ a Borel set. Prove that $f^{-1}(B)$ is a measurable set. **4**

10. (A) Let $f$ be a bounded function defined on the finite interval $[a, b]$. Prove that $f$ is Riemann integrable over $[a, b]$ if, and only if it is continuous a.e. **12**

OR

(B) State and prove Fatou's lemma. **12**

11. (A) (a) If $\mu$ is a $\sigma$-finite measure on a ring $\mathscr{R}$, prove that it has a unique extension to the ring $\mathscr{S}(\mathscr{R})$. **8**

(b) Describe the ring generated by the finite open intervals. **4**

OR

(B) Let $\mu^*$ be an outer measure on $\mathscr{H}(\mathscr{R})$ and $\mathscr{S}^*$ denote the class of $\mu^*$- measurable sets. Prove that $\mathscr{S}^*$ is a $\sigma$-ring and $\mu^*$ restricted to $\mathscr{S}^*$ is a complete measure. **12**

12. (A) For $p \geq 1$, prove that $L^p(\mu)$ is a complete metric space. **12**

OR

(B) (a) Prove that every function convex on an open interval is continuous. **6**

(b) State and prove Holder's inequality. **6**

13. (A) (a) State and prove the Jordan decomposition theorem. **8**

(b) Show that if $\mu$ and $v$ are measures such that $v << \mu$ and $v \perp \mu$, then $v$ is identically zero. **4**

OR

(B) If $\{f_n\}$ is a sequence of measurable functions which is fundamental in measure, prove that there exists a measurable function $f$ such that $f_n \to f$ in measure. **12**

**(5 $\times$ 12 = 60 Marks)**

---