(Pages : 3) **T – 6372**

Reg. No. : .....

Name : .....

Second Semester M.Sc. Degree Examination, September 2024

Mathematics

MM 223 : TOPOLOGY – II

(2020-2022 Admission)

Time : 3 Hours Max. Marks : 75

PART – A

Answer any **five** questions. Each question carries **3** mark.

1. Prove that product of any family of Hausdorff spaces is a Hausdorff space.

2. Let $f : X \to Y$ be a continuous function from a space X onto a space Y. If f is either open or closed, prove that Y has a quotient topology determined by f.

3. Show that a space X is a T_1-space if and only *if* each finite subset of X is closed.

4. Prove or disprove; subspace of a normal space is normal.

5. Prove that every finite point set in a Hausdorff space is closed.

6. Is every filter $\mathcal{F}$ contained in some ultrafilter? Justify your answer.

7. Prove that $[a, b]$ is contractible to each of its point.

8. State and prove the Brouwer No Retraction theorem.

($5 \times 3 = 15$ Marks)

P.T.O.

PART – B

Answer **all** questions. Each question carries **12** marks.

9. A. (a) Show that product of a finite number of connected spaces is connected. **6**

(b) Define **6**

(i) Product topology

(ii) Box topology

(iii) Quotient topology

OR

B. (a) Prove that the Cantor set C is homeomorphic to a countable infinite product of 2-point spaces. **6**

(b) Prove that product of a finite number of compact spaces is compact. **6**

10. A. State and prove Urysohn's lemma. **12**

OR

B. (a) Show that every regular Lindelof space is normal. **6**

(b) If X is a separable normal space and E a subset of X with card $E \geq C$, show that E has a limit point in X. **6**

11. A. State and prove Tychonoff theorem; prove at least one significant result used in it. **12**

OR

B. (a) Show that a net has y as a cluster point if and only it has a subnet which converges to y. **6**

(b) Prove that the following are equivalent for a topological space X.

(i) X is Hausdorff

(ii) limits in X are unique (that is, no net or filter in X converges to more than one point)

(iii) the diagonal $\Delta = \{(x,x) / x \in X\}$ is closed in $X \times X$. **6**

12. A. (a) Prove that the fundamental group $\pi_1(S^1)$ is isomorphic to the additive group $\mathbb{Z}$ of integers. **6**

(b) For loops α, β in S^1 with the base point 1, show that $[\alpha] = [\beta]$ if and only if $\deg(\alpha) = \deg(\beta)$. **6**

OR

B. State and prove covering path property. **12**

13. A. For $n \geq 2$, show that the n-sphere S^n is simply connected. **12**

OR

B. (a) Let X_1 and X_2 be spaces with base points x_1 and x_2 respectively. Show that $\pi_1(X_1 \times X_2, (x_1, x_2)) \cong \pi_1(X_1, x_1) \otimes \pi_1(X_2, x_2)$. **6**

(b) State and prove Brouwer fixed point theorem. **6**

($5 \times 12 = 60$ Marks)

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