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Reg. No. : ....

Name : ....

Fourth Semester M.Sc. Degree Examination, July 2024

Mathematics

Elective II

MM 244.6 : SPECTRAL GRAPH THEORY

(2020 Admission Onwards)

Time : 3 Hours Max. Marks : 75

SECTION – A

Answer **any five** questions. Each question carries **3** marks.

1. Define independence number of a graph. Compute the independence number of Peterson graph.

2. Show that 0 is a simple Laplacian eigen value having constant function 1 as eigen function.

3. Find the adjacency spectrum of the *Andrasfái* graph. Compare the diameter and the number of distinct eigen values.

4. Prove or Disprove with reason : A graph whose Laplacian eigenvalues are 0, 2, 5 must be the Peterson graph.

5. Let M and N be square matrices of the same size. Then prove that MN and NM have the same eigenvalues.

6. Prove that the regularity of a graph can be read off from its adjacency spectrum.

7. State and prove Will's upper bound for the chromatic number of a graph.

8. Show that the half cubed graph $\frac{1}{2}Q_n$ has isoperimetric constant $n-1$.

(5 × 3 = 15 Marks)

P.T.O.

SECTION – B

Answer **all** questions. Each question carries **12** marks.

9. (A) (i) Prove that the graph is bipartite if and only if its adjacency spectrum is symmetric about 0.

   (ii) Find the adjacency and Laplacian spectrum of the complete graph K_n.

OR

   (B) (i) State and prove Brooks theorem on the chromatic number of a graph.

   (ii) Prove that the adjacency eigenvalues lie in the interval $[-d, d]$ and the Laplacian eigenvalues lie in the interval $[0, 2d]$.

10. (A) (i) Find all the adjacency eigenvalues of the Cayley graph of a finite abelian group G with respect to a symmetric generating subset not containing the identity.

   (ii) If a regular graph has diameter 2 and girth 5, then prove that the size and the degree are one of the following : $n = 5$ and $d = 2$; $n = 10$ and $d = 3$; $n = 50$ and $d = 7$; $n = 3250$ and $d = 57$.

OR

   (B) (i) Compute the expressions of the distinct eigenvalues of strongly regular graphs (with multiplicities) in terms of its parameters (n, d, a, c).

   (ii) Prove that a regular graph having three distinct adjacency eigenvalues is strongly regular.

11. (A) Prove that the Laplacian eigenvalues $\lambda_{\max}$ is simple and it has an alternating eigenfunction.

OR

   (B) Prove that the adjacency eigenvalue $\alpha_{\max}$ is simple, and it has a positive eigenfunction.

12. (A) (i) Let X' be a graph obtained by removing an edge from the graph X. Then prove that the adjacency and the Laplacian eigenvalues of X' can be bounded respectively as

$$\alpha_k - 1 \leq \alpha' k \leq \alpha_k + 1$$
$$\lambda_k - 2 \leq \lambda' k \leq \lambda_k$$

(ii) Prove that the following inequality always holds; $d_{\min} \leq \alpha_k + \lambda_k \leq d$.

OR

(B) Prove that for a graph of diameter δ

$$\lambda_{k+1} \leq d - 2\sqrt{d-1} \cos \frac{2\pi k}{\delta} \text{ for all } 1 \leq k \leq \frac{\delta}{2}.$$

13. (A) (i) Show that for a regular graph G, the independence number is bounded above by $\frac{n}{1-(d/\alpha)}, \chi \geq 1-(d/\alpha)$ where α denotes $\alpha_{\min}$.

(ii) Show that the isoperimetric constant is bounded below using spectral methods.

OR

(B) State and prove the upper bound of the isoperimetric constant β in terms of maximal degree d and smallest non-trivial eigenvalue of Laplacian, λ_2.

(5 $\times$ 12 = 60 Marks)
