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Reg. No. : .....

Name : .....

**First Semester B.Sc. Degree Examination, May 2024**

**First Degree Programme under CBCSS**

**Mathematics**

**Core Course**

**MM 1141 : METHODS OF MATHEMATICS**

**(2023 Admission)**

**Special Examination**

Time : 3 Hours

Max. Marks : 80

SECTION – I

**All** the first **ten** questions are **compulsory**. They carry **1** mark each.

1. When we say  $f$  is decreasing on an interval?
2. A function  $f$  is concave up on  $(a, b)$  if  $f'$  is....
3. Determine whether the statement is true or false: Assume that  $f$  is differentiable everywhere. If  $f$  is increasing on  $[0, 2]$ , then  $f(2) > f(1) > f(0)$ .
4. If  $f'(x_0) = 0$  and  $f''(x_0) < 0$ , then  $f$  has a .... at  $x_0$ .
5. If  $f$  has an absolute extremum on an open interval  $(a, b)$ , then it must occur at a .... point of  $f$ .
6. Give an example of logical statement.

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7. Define tautology.
8. State the well – ordering principle.
9. State true or false: There are infinitely many primes.
10. If  $(a, p) = 1$ , then  $(a + rp, p) = \dots$

**(10 × 1 = 10 Marks)**

## SECTION – II

Answer any **eight** questions. These questions carry **2** marks each.

11. Find  $x$  such that  $\log x = \sqrt{2}$ .
12. Let  $y = 5 - x^2$ . Find  $dy$  and  $\Delta y$  at  $x = 2$  with  $dx = \Delta x = 0.1$
13. Find the intervals on which  $f(x) = x^2 - 4x + 3$  is increasing and the intervals on which it is decreasing.
14. Let  $s(t) = 7t - 2t^2$  be the position function of a particle moving along an  $s$ -axis, where  $s$  is in meters and  $t$  in seconds. Find the acceleration at time  $t = 3$ .
15. State the Rolle's theorem.
16. Find the critical points of  $f(x) = x^3 - 3x + 3$ .
17. When we say that a function  $f$  is concave down? Give an example.
18. Write an example of conditional statement.
19. Construct a truth table for the compound statement  $(p \vee q) \rightarrow (p \wedge q)$ .
20. State Euclid's algorithm.
21. Find seven consecutive integers  $< 100$  that are composite numbers.
22. Find the canonical decomposition of 2520.

**(8 × 2 = 16 Marks)**



### SECTION – III

Answer any **six** questions. These question carry **4** marks each.

23. Find the relative extrema of  $f(x) = 3x^5 - 5x^3$ .
24. Determine by inspection whether  $p(x) = 3x^4 + 4x^3$  has any absolute extrema. If so, find them and state where they occur.
25. Show that the function  $f(x) = (x^3 / 4) + 1$  satisfies the hypotheses of the Mean-Value Theorem over the closed interval  $[0, 2]$  and find all values of  $c$  in the interval  $(0, 2)$  at which the tangent line to the graph of  $f$  is parallel to the secant line joining the points  $(0, f(0))$  and  $(2, f(2))$ .
26. A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 100 running metre of chicken wire is available for the fence?
27. State pigeon hole principle. Show that if eleven positive integers are chosen, two of them will have the same remainder when divided by ten.
28. Find the number of positive integers  $\leq 2000$  and divisible by 2, 3, or 5.
29. Show that every integer  $n \geq 2$  has a prime factor.
30. Write the negation of the following statements.
  - (a) If I drive then I will not walk.
  - (b) He swims if and only if the water is warm.
  - (c) Some birds can talk.
  - (d) Everyone in the class has good attitude.
31. Prove that  $\sim(p \wedge q) \Leftrightarrow [(\sim p) \vee (\sim q)]$  using truth tables.

**(6 × 4 = 24 Marks)**



## SECTION – IV

Answer any **two** questions. **These** question carry **15** marks.

32. (a) The diameter of a polyurethane sphere is calculated with percentage error within  $\pm 0.4\%$ . Estimate the percentage error in the calculated volume of the sphere.
- (b) Use the first and second derivatives of  $f(x) = (2x+1)^3$  to find the intervals on which  $f$  is increasing, decreasing, concave up, concave down and the  $x$  – co-ordinates of all inflection points.
33. (a) An open box is to be made from a 16 inch by 30 inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. What size should the squares be to obtain a box with the largest volume?
- (b) Find the absolute maximum and absolute minimum values of the function  $f(x) = 2x^3 - 15x^2 + 36x$  on the interval  $[0, 5]$  and determine where these values volume.
34. (a) Let  $a$  and  $b$  are integers,  $b > 0$ , prove that there exist unique integers  $q$  and  $r$  such that  $a = b.q + r$  where  $a \leq r < b$ .
- (b) Using Euclidean algorithm, find  $(2210, 493)$
35. Explain any three proof techniques with one example each.

**(2 × 15 = 30 Marks)**

