(Pages : 6) **M – 2339**

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme under CBCSS

Mathematics

Foundation Course II

MM 1221 : FOUNDATIONS OF MATHEMATICS

(2020 Admission Regular)

Time : 3 Hours Max. Marks : 80

PART – A

All the **first** 10 questions are compulsory and each carries **1** mark :

1. Write the negation of the statement

   "M is a cyclic subgroup"

2. Indicate whether the statement

   "5 is not prime or 8 is prime" is true or false.

3. Rewrite the statement "There exists a number less than 7" using $\exists$, $\forall$ and $\ni$, as appropriate.

4. If A, B, C are subsets of a universal set U, then state whether the statement

   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ is true or false.

5. Write the parametric equation of semicubical parabola.

P.T.O.

6. Write the equation of parabola with focus (p, o) and directrix $x = -p$.

7. Which logical connective corresponds to the set relationship $A \subseteq B$?

8. Find the dot product of the vectors $< 3, 5 >$ and $< -1, 2 >$.

9. What is the general form of equation of a plane?

10. Describe the surface $z = (x-1)^2 + (y+2)^2 + 3$.

PART – B

Answer **any eight** questions from questions 11 to 26. These questions carry **2** marks :

11. Write the truth table for $p \vee q$.

12. Identify the antecedent and consequent in the statement "If n is an integer, then $2n$ is an even integer".

13. Provide a counter example to the statement "Every Continuous function is differentiable".

14. Write the contrapositive statement of the statement "continuity is a necessary condition for differentiability".

15. Let $f : A \to B$ and $g : B \to C$ be bijective. Then prove that $g \circ f : A \to C$ is bijective.

16. State the reflection property of parabolas.

17. Determine a rotation angle θ that will eliminate the xy-term in the equation $2x^2 + xy + 2y^2 + x - y = 0$.

18. Name the conic for which the set of points whose distance to the point (2, 3) is half the distance to the line $x + y = 1$.

19. Find the new coordinates of the point (2, 4) if the coordinate axes are rotated through an angle of $\theta = 30^\circ$.

20. Find the equation of the hyperbola with vertices $(0, \pm 8)$ and asymptotes $y = \pm \frac{4}{3} x$.

21. Find the arc length of the spiral $r = e^{\theta}$ between $\theta = 0$ and $\theta = \pi$.

22. Find the parametric equations of the line passing through $(4, 2)$ and parallel to $v = \langle -1, 5 \rangle$.

23. Calculate the scalar triple product $u \cdot (v \times w)$ of the vectors $u = 3\hat{i} - 2\hat{j} - 5\hat{k}$, $v = \hat{i} + 4\hat{j} - 4\hat{k}$, $w = 3\hat{j} + 2\hat{k}$.

24. Show that $u \times u = 0$ for any vector u in 3-space.

25. Find the vector of length 2 that makes an angle of $\frac{\pi}{4}$ with the positive x-axis.

26. Find the distance d between the points $(2, 3, -1)$ and $(4, -1, 3)$.

PART – C

Answer **any six** questions from questions 27 to 38. These questions carry **4** marks each :

27. Construct a truth table for the compound statement

$$\sim(p \wedge q) \Leftrightarrow [(\sim p) \vee (\sim q)]$$

28. Use a truth table to verify that $p \Rightarrow q$ and $\sim q \Rightarrow \sim p$ are logically equivalent.

29. Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6\}$ be subsets of the universal set $U = \{1, 2, 3, 4, 5, 6\}$. Then what is $A \cup B$, $A \cap B$, $A \setminus B$ and $U - B$?

30. Let $A = \{1, 2, 3\}$ and $B = \{2, 4, 6, 8\}$. Which of the following relations are functions between A and B?

(a) $\{(1, 2), (2, 6), (3, 4), (2, 8)\}$

(b) $\{(1, 4), (3, 8)\}$

(c) $\{(1, 6), (2, 6), (3, 2)\}$

(d) $\{(1, 8), (2, 2), (3, 4)\}$.

31. Find the slope of the tangent iine to the circle $r = 4\cos\theta$ at the point where $\theta = \frac{\pi}{4}$.

32. Find the area of the region that is inside the cardioid $r = 4 + 4\cos\theta$ and outside the circle $r = 6$.

33. Find an equation of the parabola that is symmetric about the y-axis, has its vertex at the origin and passes through the point (5, 2).

34. Find the equations of the paraboloid $z = x^2 + y^2$ in cylindrical and spherical coordinates.

35. Find the spherical coordinates of the point that has rectangular coordinates $(x, y, z) = (4, -4, 4\sqrt{6})$.

36. Sketch the graph of the parabola $x^2 = 12y$.

37. Describe the surface $z = -(x^2 + y^2)$.

38. The planes $x + 2y - 2z = 3$ and $2x + 4y - 4z = 7$ are parallel since their normals $<1, 2, -2>$ and $<2, 4, -4>$ are parallel vectors. Find the distance between these planes.

PART – D

Answer any two questions from questions 39 to 44. These questions carry 15 marks each :

39. Find examples of relations with the following properties.

    (a) Reflexive, but not symmetric and not transitive.

    (b) Symmetric, but not reflexive and not transitive.

    (c) Transitive but not reflexive and not symmetric.

    (d) Reflexive and symmetric but not symmetric.

    (e) Reflexive and transitive but not symmetric.

40. (a) Let $f : A \to B$ and $g : B \to C$. Using the ordered pair definition of the composition $g \circ f$, prove that $g \circ f$ is a function and that $g \circ f : A \to C$.

    (b) Find an example of functions $f : A \to B$ and $g : B \to C$ such that

    (i) f and $g \circ f$ are both injective but g is not injective.

    (ii) g and $g \circ f$ are both surjective but f is not surjective.

    (iii) $g \circ f$ is bijective but neither f nor g is bijective.

41. (a) Without eliminating the parameter, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at (1, 1) and (1, –1) on the semicubical parabola $x = t^2$, $y = t^3$ $(-\infty < t < \infty)$.

    (b) In a disastrous first flight, an experimental paper airplane follows the trajectory of a particle :

    $x = t - 3\sin t$, $y = 4 - 3\cos t$ $(t \geq 0)$

    but crashes into a wall at time t = 10.

    (i) At what times was the airplane flying horizontally?

    (ii) At what time was it flying vertically?

42. Sketch the graph of $r = \cos 2\theta$ in polar coordinates, showing step by step the variation of θ as follows :

$$0 \le \theta \le \frac{\pi}{4}, \frac{\pi}{4} \le \theta \le \frac{\pi}{2}, \quad \frac{\pi}{2} \le \theta \le \frac{3\pi}{4}, \quad \frac{3\pi}{4} \le \theta \le \pi, \ \pi \le \theta \le \frac{5\pi}{4}, \quad \frac{5\pi}{4} \le \theta \le \frac{3\pi}{2},$$
$$\frac{3\pi}{2} \le \theta \le \frac{7\pi}{4}, \frac{7\pi}{4} \le \theta \le 2\pi.$$

43. (a) Find the angle between the vectors $u = \hat{i} - 2\hat{j} + 2\hat{k}$ and $v = -3\hat{i} + 6\hat{j} + 2\hat{k}$.

(b) Find the angle between a diagonal of a cube and one of its edges.

(c) Let $v\langle 2, 3\rangle$, $e_1 = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$ and $e_2 = \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$. Find the scalar components of v along e_1 and e_2 and the vectors components of v along e_1 and e_2.

44. (a) Find parametric equations of the line L passing through the points $p_1(2, 4, -1)$ and $p_2(5, 0, 7)$.

(b) Let L_1 and L_2 be the lines

$L_1 : x = 1 + 4t,\ y = 5 - 4t,\ z = -1 + 5t$

$L_2 : x = 2 + 8t,\ y = 4 - 3t,\ z = 5 + t$

Do the lines intersect?

Set II

M 2339

Second Semester B.Sc. Degree Examination 2021

First Degree Programme under CBCSS

MATHEMATICS

Foundation Course II

MM 1221: FOUNDATIONS OF MATHEMATICS

[2018 admission onwards]

Time : 3 hours Marks:80

(Scheme of valuation)

PART - A

1. M is not a cyclic subgroup. (1m)
2. False (1m)
3. $\exists x \ni x < 7$. (1m)
4. True (1m)
5. $x = t^2, y = t^3 \ (-\infty < t < \infty)$ (1m)
6. $y^2 = 4px$. (1m)
7. $x \in A \Rightarrow x \in B$. (1m)
8. 7 (1m)
9. $ax + by + cz + d = 0$. (1m)
10. Paraboloid (1m)

PART - B

11.

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

(2m)

12. Antecedent - n is an integer $\longrightarrow$ (1m)

Consequent - $2n$ is an even integer $\longrightarrow$ (1m)

13. For correct example $\longrightarrow$ (2m)

14. If a function is continuous, then it is differentiable.
$\longrightarrow$ (2m)

15. Using the results:
If f and g are surjective, then gof is surjective $\longrightarrow$ (1m)
If f and g are injective, then gof is injective. $\longrightarrow$ (1m)

16. For statement $\longrightarrow$ (2m)

17. $\cot 2\theta = \frac{A-C}{B} = \frac{2-2}{1} = 0$ $\longrightarrow$ (1m)

$\theta = \pi/4$ $\longrightarrow$ (1m)

18. Ellipse $\longrightarrow$ (2m)

19. $x' = \sqrt{3}+2$ $\longrightarrow$ (1m)

$y' = -1+2\sqrt{3}$ $\longrightarrow$ (1m)

20. For getting $\frac{y^2}{64} - \frac{x^2}{36} = 1$ $\longrightarrow$ (2m)

21. $L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$ $\longrightarrow$ (1m)

For getting $L = \sqrt{2}(e^{\pi}-1)$ $\longrightarrow$ (1m)

22. For getting $x = 4-t$ $\longrightarrow$ (1m)
For getting $y = 2+5t$ $\longrightarrow$ (1m)

23.

$$u\cdot(v\times w) = \begin{vmatrix} 3 & -2 & -5 \\ 1 & 4 & -4 \\ 0 & 3 & 2 \end{vmatrix} \longrightarrow \text{1m}$$

$$= \underline{\underline{49}} \longrightarrow \text{1m}$$

24. For showing $u\times u = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ u_1 & u_2 & u_3 \end{vmatrix} = 0 \longrightarrow \text{2m}$

25. $v = 2\cos\frac{\pi}{4}\hat{i} + 2\sin\frac{\pi}{4}\hat{j} \longrightarrow \text{1m}$

$= \sqrt{2}\hat{i} + \sqrt{2}\hat{j} \longrightarrow \text{1m}$

26. $d = \sqrt{(4-2)^2 + (-1-3)^2 + (3+1)^2} \longrightarrow \text{1m}$

$= \underline{\underline{6}} \longrightarrow \text{1m}$

PART-C

27.

p	q	$p\wedge q$ (1m)	$\sim(p\wedge q)$ (1m)	$[(\sim p)\vee(\sim q)]$ (1m)	$\sim(p\wedge q)\Longleftrightarrow[(\sim p)\vee(\sim q)]$ (1m)
T	T	T	F	F	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	T	T

28.

p	q	$p\Rightarrow q$ (1m)	$\sim p$	$\sim q$	$\sim q\Rightarrow\sim p$ (1m)	$p\Rightarrow q\Longleftrightarrow(\sim q\Rightarrow\sim p)$ (2m)
T	T	T	F	F	T	T
T	F	F	F	T	F	T
F	T	T	T	F	T	T
F	F	T	T	T	T	T

29 $A \cup B = \{1, 2, 3, 4, 6\} \longrightarrow$ (1m)

$A \cap B = \{2, 4\} \longrightarrow$ (1m)

$A \setminus B = \{1, 3\} \longrightarrow$ (1m)

$U \setminus B = \{1, 3, 5\} \longrightarrow$ (1m)

30. (a), (b) Not a function
(~~a~~), (c), (d) - are functions } $1+1+1+1 = 4m$

31. $\frac{dy}{dx} = -\frac{\cos^2\theta - \sin^2\theta}{2\sin\theta\cos\theta} \longrightarrow$ (2m)

$\frac{dy}{dx} = -\cot 2\theta \longrightarrow$ (1m)

$\left.\frac{dy}{dx}\right|_{\theta = \pi/4} = -\cot \pi/2 = 0 \longrightarrow$ (1m)

32.

$A = 2\int_0^{\pi/3} \frac{1}{2}\left[(4+4\cos\theta)^2 - 36\right] d\theta \longrightarrow$ (2m)

$= 2(9\sqrt{3} - 2\pi) \longrightarrow$ (1m)

$= 18\sqrt{3} - 4\pi \longrightarrow$ (1m)

33. Since symmetric about Y-axis, either $x^2 = 4py$ ~~or $x^2 = -4py$~~.
It ~~must open up since it passes through (5, 2).~~ $\longrightarrow$ (1m)
∴ ~~eqn is of the form $x^2 = 4py$.~~ $\longrightarrow$ ~~(1m)~~
Also, $4p = \frac{25}{2}$ $\longrightarrow$ (2m)

∴ $x^2 = \frac{25}{2}y$ is the eqn. $\longrightarrow$ (1m)

34. Cylindrical : $z = r^2 \longrightarrow$ (2m)

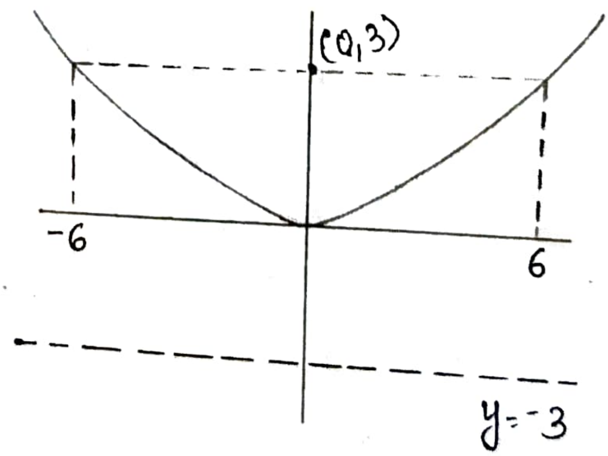
Spherical : $\rho = \cos\phi \csc^2\phi \longrightarrow$ (2m)

35. $\rho = \sqrt{x^2+y^2+z^2} = 8\sqrt{2} \longrightarrow$ (1m)

$\tan\theta = \dfrac{y}{x} = -1 \longrightarrow$ (1m)

$\cos\phi = \dfrac{z}{\sqrt{x^2+y^2+z^2}} = \dfrac{4\sqrt{6}}{8\sqrt{2}} = \dfrac{\sqrt{3}}{2} \longrightarrow$ (2m)

36. $\left.\begin{array}{l} 4P = 12 \Rightarrow P = 3 \\ \text{Directrix } y = -3 \\ 2P = 6 \end{array}\right\} \longrightarrow$ (2m)



$\longrightarrow$ (2m)

37. $z = -(x^2+y^2)$ can be written as $-z = x^2+y^2$, which can be obtained by replacing z with $-z$ in the equation $z = x^2+y^2$.

Since, the graph of $z = x^2+y^2$ is a circular paraboloid opening in the +ve z-direction, it follows that $z = -(x^2+y^2)$ is a circular paraboloid opening in the -ve z-direction. (4m)

OR

Give full mark for drawing the surface.

~~38. For obtaining~~ $\cancel{D = \frac{1}{6}} \quad \cancel{\Rightarrow (1m)}$

38. $x + 2y - \frac{z}{2} = 3$

$x + 2y - 2z = 7/2$ (1)

$$\text{Distance} = \frac{7/2 - 3}{\sqrt{1+4+4}} \quad (2)$$

$$= \frac{1}{6} \quad (1)$$

PART - D

39. For each correct example, 4 marks each — 4×4 ~~$3\times5 = 15$~~ [Maximum 15]

40. For proving (a) $\longrightarrow$ 3m

For giving correct example for (i), (ii), (iii), 4m each $4+4+4=12$

41. a) $\frac{dy}{dx} = \frac{3}{2}t \rightarrow$ 2m : $\left.\frac{dy}{dx}\right|_{t=1} = \frac{3}{2} \longrightarrow$ 1m

$\frac{d^2y}{dx^2} = \frac{3}{4t} \rightarrow$ 2m : $\left.\frac{d^2y}{dx^2}\right|_{t=1} = \frac{3}{4} \longrightarrow$ 1m

$\left.\frac{dy}{dx}\right|_{t=-1} = \frac{-3}{2} \rightarrow$ 1m $\left.\frac{d^2y}{dx^2}\right|_{t=-1} = \frac{-3}{4} \longrightarrow$ 1m

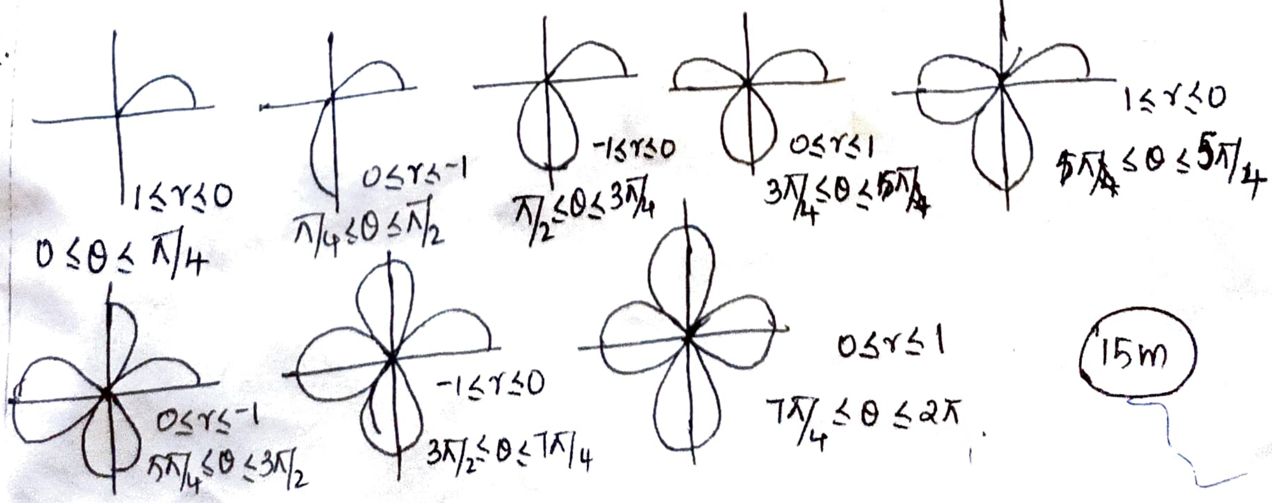
b) $\frac{dy}{dt} = 3\sin t \longrightarrow$ 1m : $\frac{dx}{dt} = 1 - 3\cos t \longrightarrow$ 1m

For obtaining time at which airplane was flying horizontally as $t=0, t=\pi, t=2\pi, t=3\pi$ $\longrightarrow$ 3m

Time ~~for~~ at which it was flying vertically

$t = \cos^{-1}\left(\frac{1}{3}\right)$, k/7 $\longrightarrow$ 2m

42.



15m

43. a) $\cos\theta = \dfrac{u \cdot v}{\|u\|\|v\|} = \dfrac{-11}{21}$

$\theta = \cos^{-1}\left(\dfrac{-11}{21}\right)$. (3m)

b) Assume cube has side a.

$d = a\hat{i} + a\hat{j} + a\hat{k}$ is a diagonal of the cube.

$\cos\alpha = \dfrac{d \cdot \hat{i}}{\|d\|\|\hat{i}\|} = \dfrac{a}{\sqrt{3a^2}} = \dfrac{1}{\sqrt{3}}$. (4m)

$\alpha = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

c) Scalar components

$v \cdot e_1 = 2\left(\frac{1}{\sqrt{2}}\right) + 3\left(\frac{1}{\sqrt{2}}\right) = \frac{5}{\sqrt{2}}$

$v \cdot e_2 = 2\left(\frac{-1}{\sqrt{2}}\right) + 3\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$ (4m)

Vector components

$(v \cdot e_1)\, e_1 = \frac{5}{\sqrt{2}} \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = \left\langle \frac{5}{2}, \frac{5}{2} \right\rangle$

$(v \cdot e_2)\, e_2 = \frac{1}{\sqrt{2}} \left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle = \left\langle \frac{-1}{2}, \frac{1}{2} \right\rangle$. (4m)

44 a) Using $P_1(2,4,-1)$,

$x = 2+3t$, $y = 4-4t$, $z = -1+8t$ $\longrightarrow$ (2m)

Using P_2 ~~(5+3t)~~ $(5,0,7)$,

$x = 5+3t$, $y = -4t$, $z = 7+8t$ $\longrightarrow$ (2m)

For proving equivalency of these 2 sets of eqn., (3m)

b) Suppose L_1 and L_2 intersect at (x_0, y_0, z_0).

Then, $x_0 = 1+4t_1$, $y_0 = 5-4t_1$, $z_0 = -1+5t_1$

and $x_0 = 2+8t_2$, $y_0 = 4-3t_2$, $z_0 = 5+t_2$ → (2m)

This leads to the condition

$$\left.\begin{aligned} 1+4t_1 &= 2+8t_2 \\ 5-4t_1 &= 4-3t_2 \\ -1+5t_1 &= 5+t_2 \end{aligned}\right\} \rightarrow \text{(1m)}$$

Lines intersect if there are values t_1 & t_2 that satisfy all the above three equations and the lines do not intersect if there are no such values → (1m)

For proving there is no such value → (3m)

∴ the lines do not intersect. → (1m)

x———x

Approved

[illegible]

Chairman

S_2 B.Sc. (CBCSS)