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Reg. No. : ..............................

Name : ..................................

**Second Semester B.Sc. Degree Examination, February 2022**

**First Degree Programme under CBCSS**

**Mathematics**

**Complementary Course for Physics**

**MM 1231.1 : MATHEMATICS II – CALCULUS WITH APPLICATIONS IN PHYSICS – II**

**(2020 Admission)**

Time : 3 Hours Max. Marks : 80

PART – A

All **ten** questions are compulsory. Each question carry **1** mark.

1. Find the complex conjugate of $3+2i+3i$.

2. Evaluate $Re(\exp(2iz))$.

3. Find the total differential of the function $f(x, y) = x^4 \exp(y)$.

4. Check whether $xdy + ydx$ is exact or not?

5. Write down the necessary condition for a stationary point of the function $f(x, y)$ is minimum.

6. Find the Jacobian of the transformation, $x = 4u - 3v^2$, $y = u^2 - 6v$.

7. Evaluate $\int_0^1\int_0^1\int_0^1 dz\,dx\,dy$.

8. Find derivative of $r(t) = t^2 i + e^t j + (2\cos \pi t)k$.

9. Find gradient of $f(x, y) = (x + y^2)$.

10. Write down the formula for finding divergence of vector field $f(x, y, z)$.

P.T.O.

## PART – B

Answer **any eight** questions. Each question carries 2 marks.

11. Find the value of $z = i^{i}$.

12. Find the modulus and argument of the complex number $z = 2 - 3i$.

13. Express $z = \dfrac{3-2i}{-i+4i}$ in terms of $x + iy$.

14. Find $f_x(3, 1)$ for the function $f(x, y) = \sin(y^2 - 4x)$.

15. Show that $2xydx + (x^2 + 3y^2)dy = 0$ is exact.

16. Determine the stationary points of $f(x, y) = x^2 - 3y^2 - 2x + 6y$.

17. Show that $f(x, y) = 2x^2 + 2y^2$ has a minima at (0, 0).

18. Find $\dfrac{df}{dt}$ if $f(x, y) = 3x^2y^3$, where $x = t^4, y = t^3$.

19. Evaluate $\int_0^1 \int_2^5 xy \; dx \; dy$.

20. Write down the formula for finding Jacobian of $x, y, z$ with respect to $u, v, w$.

21. Evaluate the triple integral $\int_1^2 \int_2^3 \int_0^1 8xyz \; dz \, dx \, dy$.

22. Find the centre of mass of the solid hemisphere bounded by the surfaces $x^2 + y^2 + z^2 = 4$ and the xy-plane, assuming that it has a uniform density 1.

23. Find the divergence of the vector field $\alpha = x^2y\,i + 2y^3z\,j + 3z\,k$.

24. Find curl of the vector field $F = yzi + xy^2j + yz^2k$.

25. Show that curl $(F) = 0$, where $F = 2xi + 2yj + 2zk$.

26. The position vector of a particle at time $t$ is given by $r(t) = \cos(2t) + \sin(2t)\,j$. Find velocity of the particle.

## PART – C

Answer any **six** questions. Each question carry 4 marks.

27. Show that the hyperbolic function $\sinh^{-1} x = \ln\left(\sqrt{1+x^2}+x\right)$.

28. By writing $\frac{\pi}{12}=\frac{\pi}{3}-\frac{\pi}{4}$ and considering $e^{\frac{i\pi}{12}}$, evaluate $\cot\left(\frac{\pi}{12}\right)$.

29. Find derivative of $e^{3x}\cos 4x$ and $e^{3x}\sin 4$ with respect to $x$ by using complex exponential.

30. If $z=\tan^{-1}\left(\frac{y}{x}\right)$, prove that $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$.

31. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of $r$ and $s$ if $w=x^2+y^2$, $x=r-s$, $y=r+s$.

32. Show that $f(x, y) = xy$ has a saddle point at $(0, 0)$.

33. Evaluate $\iint_R xy\, dA$ over the region $R$ enclosed between the lines $y=\frac{x}{2}$, $y=\sqrt{x}$, $x=2$ and $x=4$.

34. Find the volume of the solid in the first octant bounded by the coordinate planes and the plane $x+y+z=1$.

35. A triangle lamina with vertices (0, 0), (0, 1) and (1, 0) has density function $\rho(x, y)=xy$. Find its total mass.

36. Find the Laplacian of scalar field $\phi = x^2y^2z^2$.

37. Show that $\nabla\cdot(\nabla\varphi\times\nabla\psi)=0$, where $\varphi$ and $\psi$ are scalar fields.

38. Find $r_\phi \times r_\theta$ where $r=\sin\phi\cos\theta\, i+\sin\phi\sin\theta\, j+\cos\phi\, k$.

PART – D

Answer **any two** questions. Each question carry **15** marks.

39. (a) Evaluate

(i) $\exp(i^3)$, (7)

(ii) $(1+i\sqrt{3})^{\frac{1}{2}}$. (8)

(b) Use de Moivre's theorem with $n = 4$ to prove that

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$$

40. (a) Find Taylor expansion up to quadratic terms of $f(x, y) = x\exp(y) + 1$ for $(x, y)$ near the point (1, 0). (7)

(b) Locate all relative extrema and saddle points of $f(x, y) = 2xy - x^3 - y^2$. (8)

41. (a) Evaluate double integral $I = \iint_R (2 + \sqrt{x^2 + y^2} dx\, dy$, where $R$ is the region bounded by the circle $x^2 + y^2 = 2^2$. (7)

(b) Find the mass of tetrahedron bounded by the three coordinate surfaces and the plane $\frac{x}{3} + \frac{y}{3} + \frac{z}{3} = 1$, if its density is given by $4\left(1 + \frac{z}{2}\right)$. (8)

42. (a) Find the velocity and position vector of the particle, if the acceleration vector $\alpha(t) = \sin t\, i + \cos t\, j + e^t k$; $v(0) = k$, $r(0) = -i + k$. (7)

(b) Find $\nabla \cdot (\nabla \times F)$ and $\nabla \times (\nabla \times F)$ where $F = e^{xz} i + 4xe^y j - e^{yz} k$. (8)

43. (a) By integrating $e^{(\alpha+ib)x}$ and separating real and imaginary parts, find the integrals of $e^{\alpha x} \cos bx$ and $e^{\alpha x} \sin bx$. (7)

(b) The temperature of a point $(x, y)$ on a unit circle is given by $T(x, y) = 1 + xy$. Find the temperature of the two hottest points on the circle. (8)

44. (a) Find an expression for a volume element in spherical polar coordinates. (7)

(b) Calculate moment of inertia of a uniform sphere of radius $\alpha$ and mass $M$. (8)

**Second Semester B.Sc. Degree Examination 2021**

**First Degree Programme under CBCSS**

**MATHEMATICS**

**Complementary Course for Physics**

**MM 1231.1: MATHEMATICS II – Calculus with Applications in Physics-II**

**[2018 admission onwards]**

**Time: 3 hours** **Marks: 80**

## SCHEME OF VALUATION

### Part A

1. $3 - i(2+3)$
2. $e^{-y}(\cos x)$
3. $[4x^3 \exp(y)]dx + [x^4 \exp(y)]dy$
4. exact
5. $f_{xx} > 0, f_{yy} > 0, \ f_{xx}f_{yy} - f_{xy}^2 > 0$
6. $J = -24 + 12uv$
7. 1
8. $r'(t) = 2ti + e^t j - (2\pi \sin \pi t)k$
9. $i + 2yj$
10. $\text{div} f = \nabla . f = f_x + f_y + f_z$

### Part B

11. $i = e^{i(\frac{\pi}{2}+2n\pi)}$ (1), $i^i = e^{-(\frac{\pi}{2}+2n\pi)}$ (1)
12. $|z| = \sqrt{13}$ (1), $\arg z = \tan^{-1}(-\frac{3}{2}), \ldots$ (1)
13. $z = \frac{(3-2i)(-1-4i)}{(-1+4i)(-1-4i)}$ (1), $z = \frac{-11}{17} - i\frac{10}{17}$ (1)
14. $f_x(x,y) = -4\cos(y^2 - 4x)$ (1), $f_x(3,1) = -4\cos 11$ (1)
15. $A(x,y) = 2xy, B(x,y) = x^2 + 3y^2$ (1), $A_y = 2x = B_x$ *exact* (1)
16. Finding $f_x = 2x - 2, f_y = -6y + 6$ (1), stationary point (1,1) (1)
17. stationary point (0,0) (1), showing (0,0) is minima (1)
18. $\frac{df}{dt} = \frac{\partial f}{\partial x}.\frac{dx}{dt} + \frac{\partial f}{\partial y}.\frac{dy}{dt}$ (1), $ans = 51t^{16}$ (1)
19. evaluating integral (1) ans= 3 (1)
20. $J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \\ \frac{\partial x}{\partial w} & \frac{\partial y}{\partial w} & \frac{\partial z}{\partial w} \end{vmatrix}$ (2)

21. Evaluating integral (1), ans= 15 (1)
22. Formula (1), $ans = \frac{6}{8}$
23. Div a =$\nabla.a$ (1), $ans = 2xy + 6y^2z + 3$ (1)
24. Curl formula (1), final answer = $z^2i + yj + (y^2 - z)k$ (1)
25. Curl formula (1), showing curl F =0 (1)
26. Velocity =$\frac{dr}{dt}$ (1), $v(t) = -2sin2t\,i + 2\,cos2t\,j$ (1)

## PART C

27. $x = sinhy$ (1), $e^y = cshy + sinhy$ (1), $e^y = \sqrt{1+x^2} + x$ (1), $y = \ln(\sqrt{1+x^2} + x)$ (1).
28. $\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ (1),
    $\sin\left(\frac{\pi}{3}\right) = \frac{1}{2}, \cos\left(\frac{\pi}{3}\right) = \left(\frac{\sqrt{3}}{2}\right)$ (1), *evaluating and* $ans = 2 + \sqrt{3}$ (2)
29. $z = e^{(3+4i)x}$ (1), $\frac{dz}{dx} = (3+4i)e^{(3+4i)x}$ (1), *equating real and img part*,
    $\frac{d}{dx}e^{3x}cos4x = e^{3x}(3cos4x - 4sin4x), \frac{d}{dx}e^{3x}sin4x = e^{3x}(4cos4x + 3sin4x)$ (2)
30. $\frac{\partial^2 z}{\partial x^2} = \frac{2xy}{(x^2+y^2)^2}$ (2), $\frac{\partial^2 z}{\partial y^2} = \frac{-2xy}{(x^2+y^2)^2}$
31. $\frac{\partial w}{\partial r} = 4r$ (2), $\frac{\partial w}{\partial s} = r + s$ (2)
32. $f_x = y, f_y = x$ (1), (0,0) is a stationary point (1), $f_{xx} = 0, f_{yy} = 0, f_{xy} = 1$ (1), Showing (0,0) is a saddle point (1).
33. $\int_2^4 \int_{\frac{x}{2}}^{\sqrt{x}} xy\,dydx$ (2), *ans:* $\frac{11}{6}$ (2)
34. $V = \iiint dV$ (1) $V = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} dzdydx$ (2), final answer =$\frac{1}{6}$ (1)
35. Mass= $\iint_R \rho(x,y)dA$ (1), Mass=$\int_0^1 \int_0^{1-x} xy\,dydx$ (2), final answer = $\frac{1}{24}$ (1)
36. Formula (1), evaluation of derivatives (2), final answer= $2y^2z^2 + 2x^2z^2 + 2y^2z^2$ (1)
37. $\nabla.(\nabla\varphi \times \nabla\psi) = \nabla\psi.(\nabla \times \nabla\varphi) - \nabla\varphi(\nabla \times \nabla\psi)$ (2), $\nabla \times \nabla\varphi = 0$ (1),
    $\nabla \times \nabla\psi = 0$ (1).
38. $r_\emptyset = cos\emptyset cos\theta i + cos\emptyset sin\theta - sin\emptyset k$ (1), $r_\theta = -sin\emptyset sin\theta i + cos\emptyset cos\theta j$ (1), $r_\emptyset \times r_\theta = \sin^2\emptyset cos\theta i + \sin^2\emptyset sin\theta j + sin\emptyset cos\emptyset\,k$ (2)

## PART D

39. (a) $\exp(i^3) = \exp(-i)$ (1), $\exp(-i) = \cos 1 - i\sin 1$ (2),
    $1 + i\sqrt{3} = 2\exp\left(i\left(\frac{\pi}{3} + 2n\pi\right)\right)$ (2), $\sqrt{\frac{3}{2}} + i\sqrt{\frac{1}{2}}, \sqrt{\frac{3}{2}} - i\sqrt{\frac{1}{2}}$ (2)
    (b) writing de Moivres's theorem (1), $(\cos\theta + i\sin\theta)^4 = \cos 4\theta + i\sin 4\theta$ (2)
    Writing binomial expansion of $(\cos\theta + i\sin\theta)^4$ (2),
    *equating real part and evaluating* (3).
40. (a) $f_{xx} = 0, f_{xy} = e^y, f_{yy} = xe^y$ (2), $f_{xx}(1,0) = 0, f_{xy}(1,0) = 1, f_{yy}(1,0) = 1$ (2),
    *formula* (1), *final answer* $f(x,y) = 1 + x + xy + \frac{y^2}{2}$ (2)

(b) $f_x = 2y - 3x^2$ (1),

$f_y = 2x - 2y$ (1), *stationary points* $(0,0), \left(\frac{3}{2}, \frac{3}{2}\right)$ (2)

*showing* $\left(\frac{3}{2}, \frac{3}{2}\right)$ *relative maxima* (2), $(0,0)$ *saddle point* (2).

41. (a) substituting $x = \rho\cos\phi, y = \rho\sin\phi$ (2), $I = \iint_{R'} (2+\rho) J d\rho d\phi$ (1),

$J = \rho$ (2), evaluating integral, ans= $\frac{40\pi}{3}$ (2)

(b) mass formula (1), $\int_0^3 \int_0^{3-x} \int_0^{3(1-\frac{y}{3}-\frac{x}{3})} 4\left(1+\frac{x}{3}\right) dzdydx$ (2 + 2 + 2)

*ans*: 135/6 (1)

42. (a) *evaluating* $v(t) = -\cos t\, i + \sin t\, j + e^t k + i$ (3),

$r(t) = -\sin t i - \cos t\, j + e^t k + t i + j - i$ (3), *final ans* (1)

(b) $(\nabla \times F) = -e^{yz} z i + e^{xz} x j + 4e^y k$ (2), $\nabla.(\nabla \times F) = 0$ (3),

*evaluating* $\nabla \times (\nabla \times F)$

$= (4e^y - x^2 e^{xz}) i + (-e^z(1+zy)) j + (e^{xz}(1+xz) + z^2 e^{yz}) k$ (3)

43 (a) $e^{(a+ib)x} = e^{ax}(\cos bx\, i \sin bx)$ (1)

, *evaluating* $\int e^{(a+ib)x} dx = \frac{e^{ax}}{a^2+b^2}(a e^{ibx} - ibe^{ibx}) + c$ (3) separating real and imaginary part (3)

(b) Lagrange multiplier equation (1), $\lambda = \frac{1}{2}, -\frac{1}{2}$ (1), $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{2}}$ (2)

$x = \mp\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{2}}$ (2) $T_{max} = \frac{3}{2}$ *at* $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{2}}$ (2)

44. (a) $x = r\sin\theta\cos\emptyset, y = r\cos\theta\cos\emptyset, z = r\cos\theta$ (1), Jacobian formula (1)

Evaluating Jacobian $J = r^2 \sin\theta$ (3), $dV = r^2 \sin\theta dr\, d\theta\, d\emptyset$ (2)

(b) $I = \rho \int x^2 + y^2\, dV$ (2),

$I = \rho \iiint (r^2 \sin^2\theta) r^2 \sin\theta\, dr\, d\theta\, d\phi$ (2)

Evaluating $I = \frac{8}{15}\pi a^5 \rho$ (4).

Approved

[signature]

Chairman.

$S_2$ B.Sc (CBCSS)