(Pages : 4) **M – 2321**

Reg. No. : .....

Name : .....

Second Semester B.A. Degree Examination, December 2021

First Degree Programme Under CBCSS

Mathematics

Complementary Course for Economics

MM 1231.5 — MATHEMATICS FOR ECONOMICS – II

(2020 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first 10 questions are compulsory. They carry 1 mark each.

1. Write the derivative of $\sin x$.
2. State product rule of differentiation.
3. Find $\frac{d}{dx}(x^2 + x + 1)$.
4. What is the geometrical meaning of derivative of a function at a point?
5. If total cost is $TC = 2x^2 + 15$, what is the marginal cost function MC?
6. State product rule of partial differentiation to find $\frac{\partial z}{\partial y}$, where $z = g(x,y) \cdot h(x,y)$.
7. Define saddle point.
8. What is a critical point?
9. Find the second derivative of $y = \cos x$.
10. Is $x + y = 1$ homogeneous?

P.T.O.

SECTION – II

Answer **any eight** questions from among the questions 11 to 26. These questions carry **2** marks each.

11. If the linear demand function $Q = -\frac{1}{3}P + 235$, find the total revenue function for the producer.

12. Find the minimum value of $y = ax + b$, where x is non-negative and a and b are positive constants.

13. If total cost function $TC = Q^2 + 5Q + 52$ and total revenue function $TR = -3Q^2 + 95Q$, find the marginal cost and marginal revenue.

14. Show that the marginal revenue can be expressed as $p + x\frac{dp}{dx}$.

15. Find the derivative of $y = \frac{x^2}{x^2 + 1}$.

16. Find the critical points of $f(x) = 3x^3 - 27x^2 + 45x + 28$.

17. Plot the graph of the function $f(x) = x^3$ for $x = -2$ to $x = 2$.

18. Find $\frac{dy}{dx}$ where $x = \cos t$ and $y = \sin t$.

19. Find the points of inflection of $f(x) = x^3 - 3x^2 + 2x$.

20. If x and y satisfy the equation $x^2 + y^2 = 1$, prove that $\frac{dy}{dx} = -\frac{x}{y}$.

21. Differentiate $y = x \cdot \log(x^2 + 1)$.

22. Find $\frac{dP}{dQ}$, where $Q = 94 - 3P$.

23. Find dy/dx for the implicit function $9x^2 - y = 0$.

24. Find the conditions for a function $f(x, y)$ to have relative maximum.

25. Find z_x at the point $x = 1$, $y = 2$, where $z = 2x^3y^4$.

26. Find dy/dx where $7x^6 + 4y^5 - 96 = 0$.

SECTION – III

Answer **any six** questions from among the questions 27 to 38. These questions carry **4** marks each.

27. Find equilibrium price and equilibrium quantity given Supply : $Q = \frac{2}{3}P + 150$ and Demand : $Q = -\frac{1}{3}P + 450$, using
    (a) equations
    (b) graphs.

28. If total cost and total revenue functions are $TC = Q^3 - 1.5Q^2 + 50Q + 425$ and $TR = 3200Q - 9Q^2$, find maximum profit.

29. Find $f'(x), f''(x), f'''(x)$ and $f^{(4)}(x)$, where $f(x) = x^4 - 5x^3 + 11x + 3$.

30. Given $z = \frac{14x}{9x - 4y}$, find the partial derivatives z_x and z_y.

31. Given $z = 4x^5 + 7xy + 8y^4$, find z_{xx} and z_{yy}.

32. An open box consists of a square base with vertical sides and has a volume of 8 cubic feet. What is are the dimensions of the box for a minimum surface area?

33. If $x = r\cos t$, $y = r\sin t$, find the second order derivative of x and y with respect to t.

34. Show that the demand curve $p = \frac{a}{x + b} - C$ is downward sloping and convex from below.

35. Write a short note on geometrical interpretation of partial derivatives.

36. Give any two applications of derivatives in business and economics.

37. If $x = r\cos\theta$, $y = r\sin\theta$, prove that $\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0$.

38. A firm's short run production function is given by $Q = 6L^2 - 0.2L^3$ where L denotes the number of workers. Find the size of the workforce that maximizes the average product of labour.

SECTION – IV

Answer **any two** questions from among the questions 39 to 44. These questions carry **15** marks each.

39. Given that total revenue $R(x) = 280x - 2x^2$ and total $C(x) = 60x + 5600$

    (a) Express profit as a function of x

    (b) Determine maximum level of profit

    (c) Sketch the graph of profit function.

40. Differentiate the following equations

    (a) $\frac{\sin x . \cos x}{x}$

    (b) $\sqrt{\frac{3+x}{2-x}}$

    (c) $x^2 + y^2 + 2x + 4y + 5 = 0$.

41. (a) If $y = x^2 + \frac{1}{x^2}$, show that $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4 = 0$.

    (b) Find $\frac{dy}{dx}$, where $x^3 + y^3 + 3xy = 0$.

42. Find the critical points and test to see if the function is at relative maximum or minimum given $z = 3x^3 + 2y^3 + 9x^2 - 12y^2 - 72x - 126y + 16$.

43. Optimize $f(x,y) = 120x - 2x^2 - xy - 3y^2 + 160y + 7$ subject to $3x + y = 480$.

44. (a) Determine the first and second order partial derivatives of $f(x,y) = 8x^2y^3$.

    (b) State Euler's theorem for homogeneous function and $z = x^2 - xy + 2y^2$.

Answer Key for Set II of 1231.5

Mathematics for Economics – II

Section I

All the first 10 questions are compulsory

They carry 1 mark each

1. cos x
2. $\frac{d}{dx}(f(x).g(x)) = f(x).g'(x) + f'(x).g(x)$
3. 2x + 1
4. Slope of tangent at that point
5. ~~6x~~ 4x
6. $\frac{\partial z}{\partial x} = g(x,y).\frac{\partial h}{\partial y} + h(x,y).\frac{\partial g}{\partial y}$
7. $f_{xx}.f_{yy} < (f_{xy})^2$ and f_{xx} and f_{yy} have opposite sign
8. Function is neither increasing or decreasing.
9. –cos x
10. No

Section II

Answer any 8 questions from among the questions 11 to 26

These questions carry 2 marks each

11. $R = P.Q$ – 1 mark

    Ans:- $R = -\frac{1}{3}P^2 + 235P$ – 1 mark

12. Starting with any suitable method. - 1 mark

    Answer: b - 1 mark

13. $MC = 2Q + 5$ – 1 mark

    $MR = -6Q + 95$ – 1 mark

14. Definition of marginal revenue – 1 mark

    Derivation – 1 mark

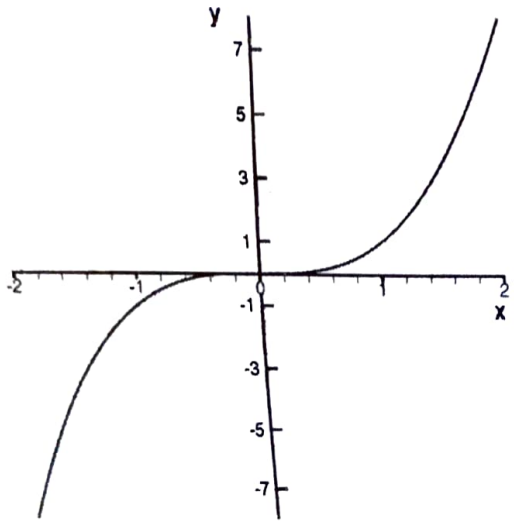
15. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x).f'(x) - f(x).g'(x)}{(g(x))^2}$ - 1 mark

$\frac{d}{dx}\left(\frac{x^2}{x^3+1}\right) = \frac{2x-x^4}{(x^3+1)^2}$ ✗ $\frac{2x}{(x^2+1)^2}$ - 1 mark

16. $f'(x) = 9x^2 - 54x + 45$ – 1 mark

Critical points x = 1, 5 – 1 mark

17. Evaluating y for different values of x - 1 mark



- 1 mark

18. $\frac{dx}{dt} = -\sin t, \frac{dy}{dt} = \cos t$ - 1 mark

$\frac{dy}{dx} = -\cot t$ - 1 mark

19. Finding derivatives - 1 mark

Answer: x = 1. - 1 mark

20. Differentiating the equation to obtain $2x + 2y\frac{dy}{dx} = 0$ – 1 mark

Proving $\frac{dy}{dx} = -\frac{x}{y}$ - 1 mark

21. $\frac{d}{dx}(f(x).g(x)) = f(x).g'(x) + g(x).f'(x)$ - 1 mark

Ans :- $\log(x^2+1) + \frac{2x^2}{x^2+1}$ - 1 mark

22. dQ/dP = -3 - 1 mark

$\frac{dP}{dQ} = -\frac{1}{3}$ - 1 mark

23. $dy/dx = -f_x/f_y$ - 1 mark
Answer: 18x - 1 mark

24. $f_x = f_y = 0$ - 1 mark
$f_{xx}, f_{yy} < 0$, $f_{xx}.f_{yy} > (f_{xy})^2$ - 1 mark

25. $z_x = 6x^2y^4$ - 1 mark
z_x at (1,2) = 96 - 1 mark

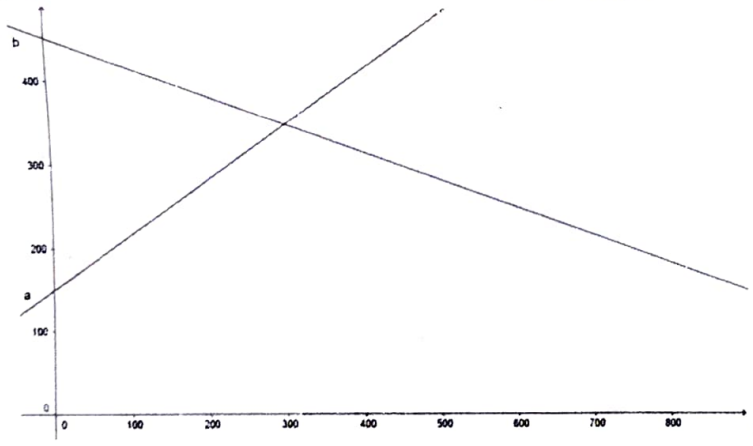
26. $f_x = 42x^5$ and $f_y = 20y^4$ - 1 mark
$dy/dx = -21x^5/10y^4$ - 1 mark

Section III

Answer any 6 questions from among the questions 27 to 38
These questions carry 4 marks each

27.(a) At equilibrium $Q_s = Q_d$ $\frac{2}{3}P + 150 = -\frac{1}{3}P + 450$ – 1mark
$P_e = 300$ and $Q_e = 350$ $\Rightarrow P = 300$ - 1 mark
$Q = 350$.

(b)



- 2 marks

28. Profit function $-Q^3 - 7.5Q^2 + 3150Q - 425$ – 1mark
Critical values Q = 30 and Q = -35 – 2 marks
Maximum profit = 60325 – 1 mark

29. $f'(x) = 4x^3 - 15x^2 + 11$ - 1 mark
$f''(x) = 12x^2 - 30x$ - 1 mark

$f''(x) = 48x - 30$ - 1 mark

$f^{(4)}(x) = 48$ - 1 mark

30. $z_x = \frac{-56y}{(9x-4y)^2}$ - 2 marks

$z_y = \frac{56x}{(9x-4y)^2}$ - 2 marks

31. $z_x = 20x^4 + 7y$ - 1 mark

$z_y = 7x + 32y^3$ - 1 mark

$z_{xx} = 80x^3$ - 1 mark

$z_{yy} = 96y^2$ - 1 mark

32. Mathematical formulation of the problem - 2 marks

Finding derivatives and simplification - 1 mark

Answer: 20 square feet. - 1 mark

33. 1st order derivatives (1 mark each) - 2 marks

2nd order derivatives (1 mark each) - 2 marks

34. Finding derivatives. - 2 marks

Conclusions. - 2 marks

35. For correct explanation give full marks. - 4 marks

36. 2 marks each for any correct application. - 4 marks

37. Finding 1st order partial derivatives. - 2 marks

Finding 2nd order partial derivatives and concluding - 2 marks

38. Finding average output function - 1 mark

Finding derivatives and further calculations - 2 marks

Answer: L = 15 - 1 mark

Section IV

Answer any 2 questions from among the questions 39 to 44

These questions carry 15 marks each

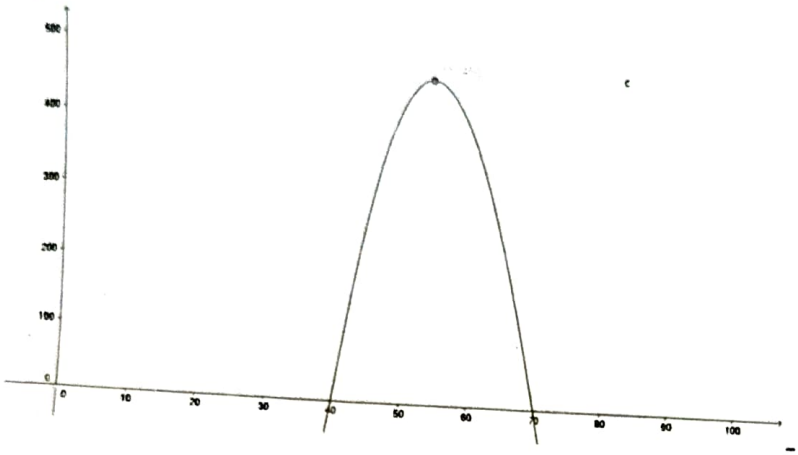
39.(a) Profit = R(x) – C(x) - 1 mark

Profit function = $-2x^2 + 220x - 5600$ - 4 marks

(b) Solving for x and finding x = 55 - 3 marks

Maximum profit = 450 - 2 marks

(c)



- 5 marks

40.(a) Applying quotient rule and product rule - 2 marks

Simplification - 2 marks

Answer = $\frac{xcosx^2 - xsinx^2 - sinx.cosx}{x^2}$ - 1 mark

(b) Applying chain rule - 2 marks

Simplification - 2 marks

Answer = $\frac{-(1+2x)}{2(2-x)\sqrt{(3+x)(2-x)}}$ - 1 mark

(c) Differentiating the equation throughout by x - 2 marks

Simplification - 2 marks

Answer = $\frac{-(x+1)}{y+2}$ - 1 mark

41.(a) Finding first order derivative - 2 marks

Finding second order derivative - 3 marks

Substitution and further simplification - 5 marks

(b) Differentiating the equation throughout by x - 2 marks

Simplification - 2 marks

Answer = $\frac{x^2+y}{x+y^2}$ - 1 mark

42.Finding partial derivatives - 4 marks

Getting critical points as (2,-3), (2,7), (-4,-3) and (-4,7) - 4 marks

Taking second order derivatives. - 4 marks

Conclusion. - 3 marks

43. Writing Lagrangian function - 2 marks

Finding partial derivatives - 6 marks

Getting $x_0 = 150$, $y_0 = 30$ and $\lambda_0 = -170$ - 5 marks

Relative minimum for $f(x,y)$ - 2 marks

44. (a) $f_x = 16xy^3$ - 2 marks

$f_y = 24x^2y^2$ - 2 marks

$f_{xx} = 16y^3$ - 2 marks

$f_{yy} = 48x^2y$ - 2 marks

$f_{xy} = f_{yx} = 48xy^2$ - 2 marks

(b) Correct statement - 2 marks

Verification - 3 marks

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Approved

Chairman

S_2 B.Sc. (CBCSS)