(Pages : 4) **L – 2475**

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, May 2021

First Degree Programme Under CBCSS

Mathematics

Core Course III

MM 1441 — ELEMENTARY NUMBER THEORY AND CALCULUS – II

(2019 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first ten questions are compulsory. These questions carry 1 mark each.

1. State whether the statement $2 \equiv -2 \ (mod\ 3)$ is true.
2. The linear congruence $ax \equiv 1 (mod\ m)$ has a unique solution if and only ________.
3. Determine whether 73215 is divisible by 9.
4. Determine whether N = 16, 151, 613, 924 is a square.
5. Prove that if $a \equiv b \ (mod\ m)$, then $a + c \equiv b + c \ (mod\ m)$ for any integer c.
6. Find parametric equations for the paraboloid $z = 4 - x^2 - y^2$.
7. Find the gradient field of $\phi(x, y) = 2x^2 + y$.
8. Give an example for a nonorientable surface.
9. State Green's theorem.
10. State the Fundamental Theorem of line integrals.

P.T.O.

SECTION – II

Answer **any eight** questions from among the questions 11 to 26. These questions carry **2** marks each.

11. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then prove that $a + c \equiv b + d \pmod{m}$.
12. Prove that the congruence relation is symmetric.
13. Prove that no prime of the form $4n + 3$ can be expressed as the sum of two squares.
14. State Eulers' formula.
15. Solve the linear congruence $12x \equiv 6 \pmod{7}$.
16. State Fubini's theorem.
17. Evaluate $\int_4^3 \int_2^4 (40 - 2xy)\, dy\, dx$.
18. Evaluate $\iint_R y^2 x\, dA$ over the rectangle $R = \{(x, y) : -3 \le x \le 2, 0 \le y \le 1\}$.
19. Use a double integral to find the area of the region R enclosed between the parabola $y = \frac{x^2}{2}$ and the line $y = 2x$.
20. Evaluate $\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} (x^2 + y^2)^{3/2}\, dy\, dx$.
21. Evaluate the triple integral $\int_{-1}^{2} \int_{0}^{3} \int_{0}^{2} 12xy^2 z^3\, dz\, dy\, dx$.
22. Explain Jacobian of transformation in two variables.
23. Find the curl of the vector field $F(x, y, z) = x^2 yi + 2y^3 zj + 3zk$.
24. Distinguish between del operator and Laplacian operator.
25. Evaluate $\int_C F \cdot dr$, where $\mathbf{F}(x, y) = \cos x\, \mathbf{i} + \sin x\, \mathbf{j}$ and C is the oriented curve $C : \mathbf{r}(t) = -\frac{\pi}{2}\mathbf{i} + t\,\mathbf{j} (1 \le t \le 2)$.
26. State Gauss's law for inverse-square fields.

SECTION – III

Answer **any six** questions from among the questions 27 to 38. These questions carry **4** marks each.

27. Find the remainder when 16^{53} is divided by 7.

28. Using the Pollard rho method, factor the integer 3893.

29. Using the Pollard $p-1$ method, find a nontrivial factor of $n = 2813$.

30. A positive integer a is self-invertible modulo p if and only if $a \equiv \pm 1 \ (mod\ p)$

31. State and prove Fermat's Little Theorem.

32. Evaluate $\iint_R \sin\theta \, dA$, where R is the region in the first quadrant that is outside the circle $r = 2$ and inside the cardioid $r = 2(1+\cos\theta)$.

33. Evaluate $\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2} \, dx \, dy$.

34. Find the surface area of that portion of the paraboloid $z = x^2 + y^2$ below the plane $z = 1$.

35. Evaluate the triple integral $\iiint_G 13xy^2z^3 \, dV$ over the rectangular box G defined by the inequalities $-1 \le x \le 2, \ 0 \le y \le 3, \ 0 \le z \le 2$.

36. Find the work done by the force field $\mathbf{F}(x, y) = (e^x - y^3)\mathbf{i} + (\cos y + x^3)\mathbf{j}$ on a particle that travels once around the unit circle $x^2 + y^2 = 1$ in the counterclockwise direction.

37. Evaluate the surface integral $\iint_\sigma x^2 \, dS$ over the sphere $x^2 + y^2 + z^2 = 1$.

38. Use the Divergence Theorem to find the outward flux of the vector field $\mathbf{F}(x, y, z) = 2x\mathbf{i} + 3y\,\mathbf{j} + z^2\mathbf{k}$ across the unit cube.

SECTION – IV

Answer **any two** questions from among the questions 39 to 44. These questions carry **15 marks each.**

39. (a) State and solve Mahavira's puzzle.

    (b) Find the remainder when 1! + 2! +..... + 100! is divided by 15.

40. (a) State and prove Wilson's Theorem.

    (b) State and prove Chinese Remainder Theorem.

41. (a) Find the volume of the solid enclosed between the paraboloids,

$$z = 5x^2 + 5y^2 \text{ and } z = 6 - 7x^2 - y^2$$

    (b) Use spherical coordinates to find the volume of the solid G bounded above by the sphere $x^2 + y^2 + z^2 = 1$ and below by the cone $z = \sqrt{x^2 + y^2}$.

42. (a) Find the surface area of that portion of the surface $z = \sqrt{4 - x^2}$ that lies above the rectangle R in the xy – plane whose coordinates satisfy $0 \le x \le 1$ and $0 \le y \le 4$.

    (b) Evaluate $\iint_R e^{xy}\,dA$ where R is the region enclosed by the lines $y = \frac{x}{2}$ and $y = x$ and the hyperbolas $y = \frac{1}{x}$ and $y = \frac{2}{x}$.

43. (a) Show that the vector field $\mathbf{F}(x, y) = 2xy^3\mathbf{i} + (1 + 3x^2y^2)\mathbf{j}$ is conservative Also find the Potential function $\phi(x, y)$.

    (b) Evaluate the surface integral $\iint_\sigma xz\,dS$, where σ is the part of the plane $x + y + z = 1$ that lies in the first octant.

44. Verify Stokes' Theorem for the vector field $\mathbf{F}(x, y, z) = 2z\mathbf{i} + 3x\mathbf{j} + 5y\mathbf{k}$, taking σ to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \ge 0$ with upward orientation, and C to be the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of σ in the xy – plane.
