(Pages : 4) **M – 7081**

Reg. No. : ....

Name : ....

Third Semester M.Sc. Degree Examination, March 2022

Mathematics

Elective — I

MM 233 : OPERATION RESEARCH

(2020 Admission)

Time : 3 Hours Max. Marks : 75

PART – A

Answer **any five** questions among the questions 1 to 8. Each question carries **3** marks.

1. Explain the procedure of solving LPP using graphical method.
2. Define slack, surplus and artificial variables in a LPP.
3. What are transportation problems? Explain any two methods of obtaining initial feasible solution for a transportation problem.
4. Using Vogel's approximation method, find an initial basic feasible solution of the transportation problem:

	D_1	D_2	D_3	D_4	Supply
S_1	2	3	11	7	6
S_2	1	0	6	1	1
S_3	5	8	15	9	10
Demand	7	5	3	2	

5. Discuss the various steps involved in the applications of PERT and CPM.

6. Prove that if $F(X, Y)$ has a saddle point (X_0, Y_0) for all $Y \geq 0$, then $G(X_0) \leq 0, Y'_0 \, G(X_0) = 0$.

7. Write the Kuhn-Tucker conditions for :

Minimize $f = 4(x_1 - 6)^2 + (x_2 - 2)^2$

Subject to

$$3(x_1 + 1)^2 + 6x_2 \leq 12, \; x_1, x_2 \geq 0.$$

8. Show that recursive optimization may not work even though the function is separable.

(5 × 3 = 15 Marks)

PART – B

Answer **all** questions from 9 to 13. Each question carries **12** marks.

9. (a) Use two phase simplex method to solve :

Minimize $z = x_1 - 2x_2 - 3x_3$

Subject to

$$-2x_1 + x_2 + 3x_3 = 2$$
$$2x_1 + 3x_2 + 4x_3 = 1$$
$$x_1, x_2, x_3 \geq 0$$

OR

(b) Solve the following LPP by Big-M method.

Minimize $Z = 5x_1 + 3x_2$

Subject to

$$2x_1 + 4x_2 \leq 12$$
$$2x_1 + 2x_2 = 10$$
$$5x_1 + 2x_2 \geq 10$$
$$x_1, x_2 \geq 0$$

10. (a) Solve the following transportation problem :

	P	Q	R	S	Supply
A	5	10	4	5	10
B	6	8	7	2	25
C	4	2	5	7	20
Demand	25	10	15	5	55

OR

(b) Five men are available to do five different jobs. From the past records, the time in hours that each man takes to do each job is known and is given in the following table :

		Jobs				
		I	II	III	IV	V
	A	2	9	2	7	1
	B	6	8	7	6	1
Men	C	4	6	5	3	1
	D	4	2	7	3	1
	E	5	3	9	5	1

Find out how men should be assigned the jobs in way that will minimize the total time taken.

11. (a) The following maintenance job has to be performed periodically on the heat exchangers in a refinery :

Task	Description	Immediate Predecessors	Time (Days)
A	Dismantle Pipe connections	–	14
B	Dismantle header, closure and floating head front	A	22
C	Remove tube bundle	B	10
D	Clean bolts	B	16
E	Clean header and floating head front	B	12
F	Clean tube bundle	C	10
G	Clean shell	C	6
H	Replace tube bundle	F, G	8
I	Prepare shell pressure test	D, E, H	24
J	Prepare tube pressure test and make the final reassembly	I	16

(i) Draw a network diagram of activities for the project

(ii) Identify the critical path. What is its length?

(iii) Find the total float and free float for each task.

OR

(b) Consider a project with 5 jobs A, B, C, D and E with the following job sequence: Job A precedes C and D; Job B precedes D; Job C and D precede E. The computation times for A, B, C, D and E are 3, 1, 4, 2 and 5 respectively. Construct the project network, find earlier time, latest time and slack time of each event.

12. (a) Minimize $f(x) = (x_1 + 1)^2 + (x_2 - 2)^2$

Subject to

$$g_1(X) = x_1 - 2 \leq 0$$
$$g_2(X) = x_2 - 1 \leq 0$$
$$x_1, x_2 \geq 0$$

OR

(b) Maximize $3x_1 + 6x_2 - 4x_1 x_2 - 3x_1^2 - 2x_2^2$

Subject to $3x_1 + 2x_2 \leq 4$, $x_1 + x_2 \leq 1$, $x_1, x_2 \geq 0$

13. A. (a) Write an algorithm to find shortest path in a minimum value problem.

(b) Determine max $(u_1 u_2 u_3)$ subject to $u_1 + u_2 + u_3 = 5$, $u_1, u_2, u_3 \geq 0$.

OR

B. Maximise $u_1^2 + u_2^2 + u_3^2$

Subject to $u_1 u_2 u_3 \leq 6$, $u_1 + u_2 + u_3 \leq 6$, $u_1, u_2, u_3 \geq 0$.

(5 × 12 = 60 Marks)

Approved by
(Chairman)

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MM 233 : OPERATION RESEARCH
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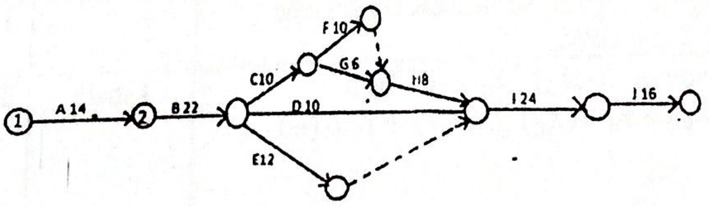
M7081

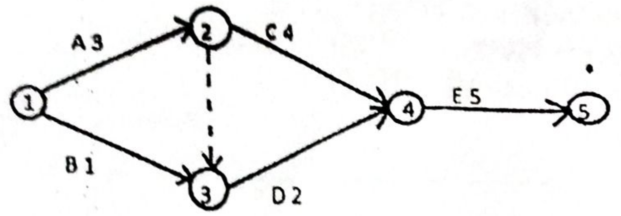
<table>
<tr><th>Qn. No</th><th></th><th>Mark distribution</th><th>Total</th></tr>
<tr><td colspan="4">PART – A
Answer any five questions among the questions 1 to 8. Each question carries 3 marks.</td></tr>
<tr><td>1</td><td>Step 1: Formulate the Linear programming problem.
Step 2: Construct a graph and plot the constraint lines.
Step 3: Identify the feasible solution region.
Step 4: Find the optimum point.</td><td>Any one step 1 mark
Rest 2</td><td>3</td></tr>
<tr><td>2</td><td>Slack variables are added to less than or equal (≤) to type constraints in order to get an equality constraint.
A surplus variable is added to greater than or equal to (≥) type constraints in order to get an equality constraint.
Artificial variables are added to those constraints with equality (=) and greater than or equal to (≥) sign. An Artificial variable is added to the constraints to get an initial solution to an LP problem.</td><td>1 mark each</td><td>3</td></tr>
<tr><td>3</td><td>The transportation problem is concerned with finding the minimum cost of transporting a single commodity from a given number of sources to a given number of destinations.
Any two methods of obtaining initial feasible solution.</td><td>1
1 mark each</td><td>3</td></tr>
<tr><td>4</td><td><table>
<tr><th></th><th>D_1</th><th>D_2</th><th>D_3</th><th>D_4</th><th></th></tr>
<tr><td>S_1</td><td>$\boxed{1}$ 2</td><td>$\boxed{5}$ 3</td><td>11</td><td>7</td><td>6</td></tr>
<tr><td>S_2</td><td>1</td><td>0</td><td>6</td><td>$\boxed{1}$ 1</td><td>1</td></tr>
<tr><td>S_3</td><td>$\boxed{6}$ 5</td><td>8</td><td>$\boxed{3}$ 15</td><td>$\boxed{1}$ 9</td><td>10</td></tr>
<tr><td></td><td>7</td><td>5</td><td>3</td><td>2</td><td></td></tr>
</table></td><td></td><td>3</td></tr>
<tr><td>5</td><td>Give full marks for writing steps involved in PERT and CPM
Or applications of PERT and CPM</td><td></td><td>3</td></tr>
<tr><td>6</td><td>Let $F(X,Y)$ have a saddle point (X_0,Y_0) , $Y_0 \geq 0$. Then
$f(X_0) + Y'G(X_0) \leq f(X_0) + Y_0'G(X_0) \leq f(X) + Y_0'G(X)$
From the left hand side
$Y'G(X_0) \leq Y_0'G(X_0)$
Reverse inequality and showing $G(X_0) \leq 0$</td><td>1 mark
2 marks</td><td>3</td></tr>
<tr><td>7</td><td>$8(x_1-6)+6y_1(x_1+1) \geq 0$
$x_1\,[8(x_1-6)+6y_1(x_1+1)]=0$
$2(x_2-2)+6y_1 \geq 0$
$x_2\,[2(x_2-2)+6y_1]=0$
$3(x_1+1)^2+6x_2-12 \leq 0$</td><td>1 mark
1 mark</td><td>3</td></tr>
</table>

	$y_1\,[3(x_1+1)^2+6x_2-12]=0$	1	
8	Taking suitable example Proof	1 2	3
	PART – B Answer all questions from 9 to 13. Each question carries 12 marks.		
9(a)	Standard form Phase -1 min $z^* = A_1 + A_2$ Simplex table 1 Simplex table 2 (solution optimal but the artificial variable A_1 appears in the basis with positive value .) Hence, feasible solution to the given original LP problem does not exist.	2 1 3 4 2	12
	or		
9(b)	Standard form Simplex table 1 Simplex table 2 Simplex table 3 Simplex table 4 Getting solution $x_1 = 4, x_2 = 1$; minimum z=23	2 2 2 2 2 2	12
10(a)	Getting initial solution Testing for optimality Modification and getting optimal solution $A \rightarrow R(10), B \rightarrow P(15), B \rightarrow R(5), B \rightarrow S(5),$ $C \rightarrow P(10) \& C \rightarrow Q(10)$ Minimum Transportation cost =235	4 4 4	12
	or		
10(b)	Row reduction Column reduction Making assignment Checking optimality and modification getting optimal solution $A \rightarrow III, B \rightarrow V, C \rightarrow I, D \rightarrow IV, E \rightarrow II$ Minimum cost 13	2 2 2 3 3	12
11(a)	 (i) (ii)Critical path ABCFHIJ , Length 104 (iii) Free float and total float	4 5 3	12

3Y2

2

H6
6

11(b)	 Table showing earlier time, latest time and slack time of each event	5 7	12
12(a)	Lagrangian function $F(X,Y) = (x_1+1)^2 + (x_2-2)^2 + y_1(x_1-2) + y_2(x_2-1)$ KT Conditions $2(x_1+1)+y_1 \geq 0 \quad x_1[2(x_1+1)+y_1] = 0$ $2(x_2-2)+y_2 \geq 0 \quad x_2[2(x_2-2)+y_2] = 0$ $x_1 - 2 \leq 0 \quad y_1(x_1-2) = 0$ $x_2 - 1 \leq 0 \quad y_2(x_2-1) = 0 \quad x_1, x_2, y_1, y_2 \geq 0$ Getting solution $x_1 = 0, x_2 = 1$; minimum $f(X) = 2$	2 3 2 1 1 3	12
	or		
12(b)	Lagrangian function KT Conditions Introducing necessary slack and surplus variables Solving the problem by quadratic programming algorithm Getting solution $x_1 = 0, x_2 = 1$	2 3 2 3 2	
13A(a)	Explaining algorithm to find shortest path with a suitable example	6	6
13A(b)	State variables $x_3 = u_1 + u_2 + u_3, x_2 = u_1 + u_2, x_1 = u_1$ $F_3(x_3) = max_{u_3}(u_3.F_2(x_2)), F_2(x_2) = max_{u_2}(u_2.F_1(x_1)),$ $F_1(x_1) = u_1$ Max $u_1u_2u_3 = \frac{125}{27}$; $u_1 = u_2 = u_3 = \frac{5}{3}$	1 2 3	6
	or		
13B	Any reasonable attempt with a single constraint.		12

Give appropriate step marks for alternate proofs and answers