(Pages : 3) **S – 6800**

Reg. No. : ....

Name : ....

Third Semester M.Sc. Degree Examination, February 2024

Mathematics

Elective II

MM 234.5 : GRAPH THEORY

(2020 Admission Onwards)

Time : 3 Hours Max. Marks : 75

SECTION – A

Answer any **five** questions. Each question carries **3** marks.

1. Show that the order of a self-complementary graph is $4k$ or $4k + 1$ for some positive integer k.

2. Draw two non-isomorphic connected graphs of order 6 but with same degree sequence.

3. Prove that $\overline{C_n}$, is Hamiltonian for $n \geq 5$.

4. Draw a graph G satisfying each of the following properties.

   (a) G is Eulerian but not hamiltonian.

   (b) G is Hamiltonian but not Eulerian.

   (c) Neither G nor $\overline{G}$ is Eulerian but both contain Eulerian trail.

P.T.O.

5. Show that the Petersen graph is not 1-factorable.

6. Draw the Turan graphs $T_{14,6}$ and $T_{21,5}$.

7. Does there exist a non-planar graph with edge chromatic number 2? Justify your answer.

8. Is the Periphery of every graph disconnected? Justify your answer.

(5×3 = 15 Marks)

SECTION – B

Answer **all** questions. Each question carries **12** marks.

9. (a) (i) Show that a graph of order at least 3 has no cut vertex if and only if every two vertices lie on a common cycle. **10**

(ii) If G is nonseparable, does $\overline{G}$ nonseparable? Justify your answer. **2**

OR

(b) (i) If G is a regular graph of degree 3, then show that $k(G) = \lambda(G)$. **9**

(ii) Give an example of a graph G for which $k(G) = 2, \lambda(G) = 3$ and $\delta(G) = 4$. **3**

10. (a) (i) Define $h(G)$ and $h^{*}(G)$. **4**

(ii) For every connected graph G, prove that $h^{*}(G) = h(G)$. **8**

OR

(b) (i) State and prove the Dirac's sufficient condition for a graph to be Hamiltonian. **10**

(ii) Show by an example that Dirac's sufficient condition is not a necessary condition for a graph to be Hamiltonian. **2**

11\. (a) State and prove a necessary and sufficient condition for a nontrivial connected digraph to be Eulerian. **12**

OR

(b) Prove that a graph G contains a 1-factor if and only if $k_o(G-S) \leq |S|$ for every $S \subsetneq V(G)$. **12**

12\. (a) (i) State and prove the Konig's theorem. **8**

(ii) Show that $\chi(G) \leq 1 + \Delta(G)$ for every graph G. **4**

OR

(b) Prove that $r(T_m, K_n) = (m-1)(n-1)+1$ for a tree T_m of order $m \geq 2$ and $n \geq 2$. **12**

13\. (a) (i) Determine which graphs of order n have the location number $n - 1$. **5**

(ii) Show that a vertex v in a connected graph G is a boundary vertex of G if and only if it is not an interior vertex of G. **7**

OR

(b) (i) Show that a non-trivial graph G is the periphery of some graph if and only if every vertex of G has eccentricity 1 or no vertex of G has eccentricity 1. **7**

(ii) Prove that $diam(G) > 3 \Rightarrow diam(\overline{G}) < 3$. **5**

(5 × 12 = 60 Marks)
